FEATURE

The (unfortunate) complexity of the economy

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The (unfortunate) complexity of the economy

Jean-Philippe Bouchaud explains how physicists are bringing new ideas and methodologies to the science of economics

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The current financial crisis puts the science of economics under huge pressure. Classical economics, which was formalized in the 1950s and the 1960s, is the theory that still shapes much of economic thinking today. Its foundations are the assumptions of economic equilibrium and rational expectations. In theory, deregulated markets should be efficient: prices faithfully reflect the underlying fundamental values, while markets ensure optimal allocation of resources. Any mispricing or forecasting error should be quickly corrected by economic agents, who act with perfect rationality and absolute knowledge about all future states of the world and their probabilities. These equilibrated markets should therefore be stable: crises can only be triggered by exogenous events – such as natural catastrophes, terrorist attacks or political disruptions – and never by the dynamics of the market itself, such as speculation or complex financial engineering. This, however, stands in contrast to most financial crashes, including the latest one, which all seem to be caused by irrational market bubbles.

Classical economics has deeply influenced scores of decision-makers high up in government agencies and financial institutions. The last 20 years of deregulation have been prompted by the argument that constraints of any kind prevent the markets from reaching their supposedly perfect efficient equilibrium state. Some of those decision-makers are now "in a state of shocked disbelief", as Alan Greenspan, the American economist and former chairman of the US Federal Reserve, declared recently. He has now admitted that he had put too much faith in the self-correcting power of free markets and had failed to anticipate the self-destructive power of wanton mortgage lending. However, a large fraction of economists still abide by the notions of economic equilibrium and rational expectations. These concepts have not only dominated economics, but also permeated international politics, sociology and law.

Unfortunately, nothing is more dangerous than dogmas donned with scientific feathers. The current crisis might offer an excellent occasion for a paradigm change, previously called for by prominent economists like John Maynard Keynes, Alan Kirman and Steve Keen. They have forcefully highlighted the shortcomings and contradictions of the classical economic theory, but progress has been slow. The task looks so formidable that some economists argue that it is better to stick with the implausible but well-corseted theory of perfectly rational agents than to venture into modelling the infinite number of ways agents can be irrational.

Physicists, however, feel uncomfortable with theories not borne out by (or even blatantly incompatible with) empirical data. But could the methodology of physics really contribute to the much-awaited paradigm shift in economics? Such an approach is called econophysics (a term coined in 1995 by Boston University physicist Gene Stanley), a field that effectively emerged from a famous 1987 conference between physicists and economists held at the Santa Fe Institute in New Mexico. After 20 years or so of "econophysics", and about 1000 papers published on the *arXiv* preprint server (a new section "Quantitative Finance" was created in December 2008), it is perhaps useful to give a personal bird's eye view of what has been achieved so far and what might be taught, in the long run, in order to foster a better grasp of the complexity of economic systems.

At a Glance: Econophysics

- The present financial meltdown reflects the underlying flaws in the current economics paradigm, which is based on the assumptions of economic equilibrium and rational expectations
- Econophysics is the application of the methods of statistical physics to economics problems and is a more empirical and intuitive approach than that taken by economists. Also, it focuses more on mechanisms and analogies, rather than on axioms and theorem proving, which is standard practice in economics
- Physicists have (re)discovered that the distribution of price changes, of company sizes, and of individual wealth, among other things, can be described by power laws. This is intriguing because many complex physical systems share the same dynamics
- Physicists have also tried to model the economy by using "toy" models inspired by physical systems, such as the random field Ising model and an approach called minority games. Although simplified, these toy models offer a more realistic perspective of the economy than traditional mainstream models

The intuition of physicists

Econophysics is in fact a misnomer, since most of its scope concerns financial markets. To some economists, finance is a relatively minor subfield and any contribution, even the most significant, can only have a limited impact on economics science at large. I personally strongly disagree with this viewpoint: recent events confirm that hiccups in the financial markets can cripple the entire economy.

From a more conceptual point of view, financial markets are an ideal laboratory for testing several fundamental concepts of economics. Are prices really such that supply matches demand? Are price moves primarily due to news? (The answer to both these questions seem to be clear "no", as I have argued elsewhere, see arXiv:0803.1769 and arXiv:0809.0822, respectively) The terabytes of data spat out everyday by financial markets allow one (in fact compel one) to compare in detail theories with observations. This proliferation of data should soon concern other spheres of economics and social science: credit cards and e-commerce

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Panic on the market
A trader on the stock exchange watches the market collapse.

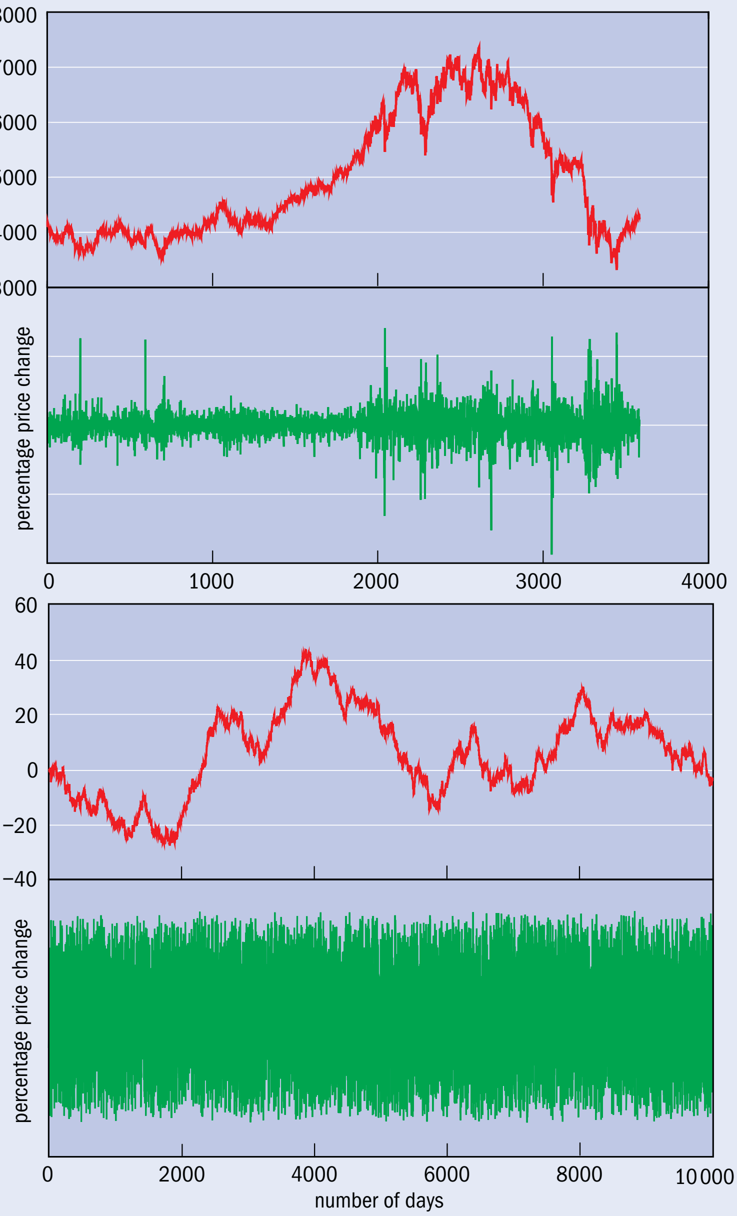
will allow one to monitor consumption in real time and to test theories of consumer behaviour in great detail (see D Sornette *et al.* 2004 *Phys. Rev. Lett.* **93** 228701). So we must get prepared to deal with huge amounts of data, and to learn to scrutinize them with as little prejudice as possible, while still asking relevant questions. These will start from the most obvious ones – those that need nearly no statistical test at all because the answers are obvious, as figure 1 exemplifies – and only then delve into more sophisticated problems. As I will try to illustrate, the very choice of the relevant questions, which ultimately leads to a deeper understanding of the data, is often sheer serendipity: more of an art than a science. That intuition, it seems to me, is well nurtured by an education in the natural sciences, where the emphasis is on mechanisms and analogies, rather than on axioms and theorem proving.

Faced with a mess of facts to explain, Richard Feynman advocated that we should choose one of them and try our best to understand it in depth, with the hope that the emerging theory will be powerful enough to explain many more observations. In the case of financial markets, physicists have been immediately intrigued by a number of phenomena described by power laws. For example, the distribution of price changes, of company sizes and of individual wealth all have a power-law tail, which is to a large extent universal. The power-law distribution of price changes goes against the popular Black–Scholes model – the financial model used to evaluate the price of equity options – that assumes fluctuations to be Gaussian. Unlike the Gaussian distribution, power laws have a long tail that accounts for rare extreme events, such as market crashes. Furthermore, the activity and volatility of markets have a power-law correlation in time, reflecting their *intermittent* nature: quiescent periods are intertwined with bursts of activity, on all timescales. Again, as figure 1 testifies, this is obvious to the naked eye.

Power laws leave most economists unruffled (aren't they, after all, just another fitting function?), but they immediately send physicists' imagination churning. The reason is that many "complex" physical systems display very similar intermittent dynamics: velocity

The activity and volatility of markets have a power-law correlation in time, but while such laws leave most economists unruffled, they immediately send physicists' imaginations churning

1 Markets: the FTSE versus a Gaussian simulation



Shown here are two representations of the time evolution of a typical market. In the FTSE index (top), the upper curve shows the price time series and the lower curve represents the daily price changes. The corresponding charts for synthetic price changes (bottom) assume the standard Black–Scholes (Gaussian) model. The difference is obvious to the naked eye, and the salient statistical features of real price changes stand out immediately by comparing the bottom graphs: large spikes correspond to large up or down market moves. The intermittent nature of the dynamics is also apparent: large moves are clustered together, suggesting analogies with intermittent dynamics observed in other complex systems, such as turbulent flow (see figure 2), Barkhausen noise (see figure 3) and crack or earthquake dynamics.

fluctuations in turbulent flows (figure 2), avalanche dynamics in random magnets under a slowly varying external field (figure 3), teetering progression of cracks in a slowly strained disordered material, and so on. The interesting point about these examples is that while the *exogenous* driving force is regular and steady, the resulting *endogenous* dynamics is complex and jittery. In these cases, the non-trivial (or as physicists say "critical") nature of the dynamics comes from collective effects: individual components have a relatively simple behaviour, but their interactions lead to new, emergent phenomena. The whole is fundamentally different from any of its elementary subparts. Since this intermittent behaviour appears to be generic for physical systems with both heterogeneities and interaction, it is tempting to think that the dynamics of financial markets, and more generally of economic systems, does reflect the same underlying mechanisms.

Modelling the economy

An example of the application of a physics toy model to economics is the random field Ising model (RFIM), a statistical-physics model that accounts for how spins order within a disordered magnet; in economics, it attempts to describe situations where there is a conflict between personal opinions, public information and social pressure (see J P Sethna *et al.* 2001 *Nature* **410** 242).

Imagine a collection of traders all having different *a priori* opinions, say optimistic (buy) or pessimistic (sell). Traders are influenced by some slowly varying global factors, such as interest rates, inflation, earnings and dividend forecasts. One assumes no shocks whatsoever in the dynamics of these exogenous factors, but posits that each trader is also influenced by the opinion of the majority. They conform to it if the strength of their *a priori* opinion is weaker than their herding tendency. So if all the agents made up their mind in isolation (zero herding tendency), then the aggregate opinion would faithfully track the external influences and, by assumption, evolve smoothly. But surprisingly, if the herding tendency exceeds some finite threshold, then the evolution of the aggregate opinion jumps discontinuously from optimistic to pessimistic as global factors deteriorate only slowly and smoothly. Furthermore, some *hysteresis* appears. Much as supersaturated vapour refuses to turn into a liquid, optimism is self-consistently maintained. In order to trigger a crash, global factors have to degrade far beyond the point where pessimism should prevail. These factors must then improve much beyond the crash tipping point before global optimism is reinstalled, again somewhat abruptly.

Although the model is highly simplified, it is hard not to see some resemblance with all bubbles in financial history. The consecutive reports about the level of leverage used by banks to pile up bad debt should have led to a self-correcting, soft landing of the global markets – or so the efficient market theory would predict. Instead, collective euphoria screened out all the bad omens until it became unsustainable. Any small, anecdotal event or insignificant news item is then enough to spark a meltdown. This exemplifies in a vivid way the breakdown of one of the cornerstones of classical economics, namely that an ensemble of heterogeneous and interacting agents can be replaced by a single "representative" one (criticized in Kirman's 1992 essay "Whom or what does the representative individual represent?", *J. Economic Perspectives* **6** 117).

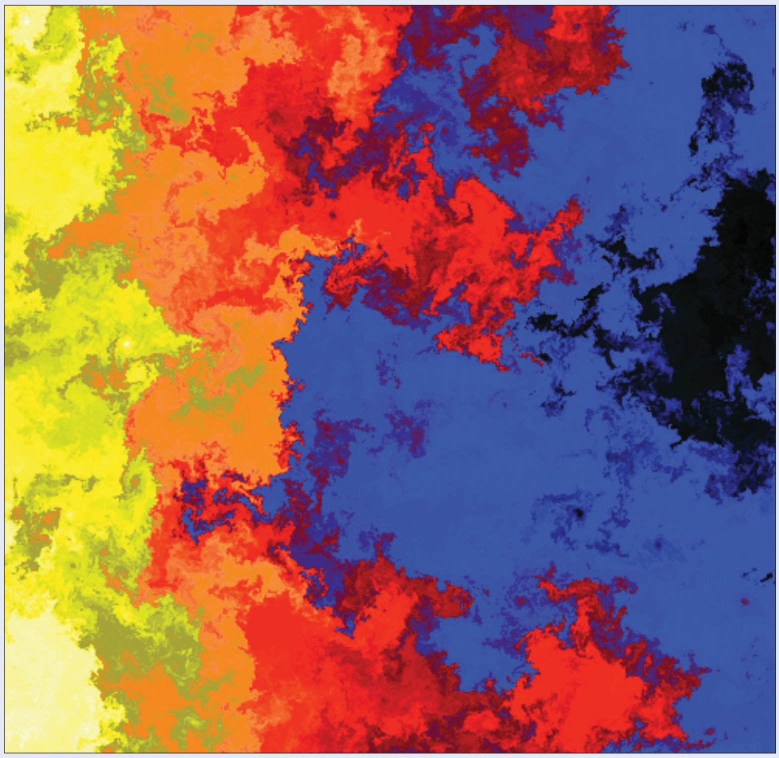
In the RFIM, this scenario is impossible: the behaviour of the crowd is fundamentally different from that of any single individual. Much as in statistical physics or materials science, the link between the micro and the macro remains one of the main theoretical challenges in economics: how, in other worlds, does one infer the aggregate behaviour (for example the aggregate demand) from the behaviour of individual elements?

Another, richer, family of models used in econophysics is called minority games, a framework in which agents learn to compete for scarce resources. A crucial aspect here is that the decisions of these agents *impact* on the market: prices move as a result of these decisions. A remarkable result is the observation, within this framework, of a genuine phase transition as the number of speculators increases. So we go from a predictable market where agents can eke out some profit from their strategies (phase 1) to an overcrowded market where these profits vanish or become too risky (phase 2). Around the critical point where predictability disappears and efficiency sets in, intermittent power-law phenomena emerge, akin to those observed on real stock markets. The cute point of this analysis is that there is a well-grounded mechanism (called "self-organized criticality" by the late Danish physicist Per Bak) that keeps the market in the vicinity of the critical point: fewer agents means more opportunities for profit, which attracts more agents; more agents means no profit opportunities, so that frustrated agents leave the market.

There are other examples, in physics and computer science, where competition and heterogeneities lead to interesting phenomena that could be metaphors for the complexity of economic systems. These include spin-glasses, the spins within which interact randomly with one another – its application to economics having been first suggested by physics Nobel laureate Philip Anderson at the 1987 Santa Fe meeting. Molecular glasses, protein folding and Boolean satisfiability problems are other examples. In all these cases, the energy of the system is an incredibly complicated function of the various degrees of freedoms (the spins, the position of the atoms of the protein, the Boolean variables). Generically, this function is found to display an exponential number (dependent on the number of degrees of freedom) of local minima, i.e. points of equilibria. The absolute best one is (a) extremely hard to find; (b) only marginally better than the next best one; and (c) extremely fragile to any change in the parameters of the problem – the best one can easily swap over to become the second best, or even cease abruptly to be a minimum. Physical systems with these "rugged" energy landscapes display characteristic phenomena that have been studied extensively in the last 20 years, both experimentally and theoretically. The dynamics is extremely slow as the system is lost amidst all these local minima; equilibrium is never reached in practice; and there is intermittent sensitivity to small changes of the environment. There is no reason to believe that the dynamics of economic systems, also governed by competition and heterogeneities, should behave very differently – at least beyond a certain level of complexity and interdependency.

If true, this would entail a major change of paradigm. First, even if an equilibrium state exists in theory, it may be totally irrelevant in practice, because the time to reach it is far too long. As Keynes noted, "in the long run we are all dead". The convergence to the "Garden of Eden" of economic systems might not be hobbled by regulations but by their tug-induced complexity. One can in fact imagine situations where regulation could nudge free, competitive markets closer to an efficient state that they would never otherwise reach. Second, complex economic systems should be inherently fragile to small perturbations, and generically evolve in an intermittent way, with a succession of rather stable epochs punctuated by rapid, unpredictable changes – again, even when the exogenous drive is smooth and steady. No big news is needed to make markets lurch wildly, in agreement with recent empirical observations (see A Joulin *et al.* 2008 arXiv:0803.1769). Within this metaphor of markets, competition and complexity could be the essential cause of their endogenous instability (on this point, see M Marsili ssrn.com/abstract=1305174).

2 Turbulence



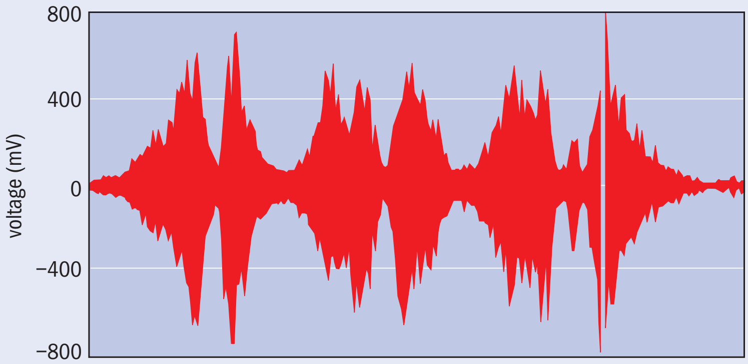
A colourful representation of a turbulent flow, showing a complex intertwining of highly active and quiescent regions (A Celani and M Vergassola 2001 *Phys. Rev. Lett.* **86** 424).

A different methodology

The above models tell interesting stories but are clearly highly stylized and aim to be inspiring rather than convincing. Still, they seem a bit more realistic than the traditional models of economics that assume rational agents with infinite foresight and infinite computing abilities. Such simplifying caricatures are often made for the sake of analytical tractability, but many of the above results can in fact be established analytically using statistical-mechanics tools developed in the last 30 years to deal with disordered systems.

One of the most remarkable breakthroughs is the correct formulation of a mean-field approximation to deal with interactions in heterogeneous systems. Whereas the simple Curie–Weiss mean-field approximation for homogenous systems is well known and accounts for interesting collective effects (see W Brock

3 Barkhausen noise



A time trace of intermittent Barkhausen noise produced by domain walls in random magnets as they unpin from impurities and repin some (random) time later. The dynamics is made up of a succession of "avalanches" of all sizes, which correspond to the domain walls sweeping areas of all size as they unlock (J P Sethna *et al.* 2001 *Nature* **410** 242).

and S Durlauf 2001 *Rev. Economic Studies* **68** 235), its heterogeneous counterpart is far subtler and has only been worked out in detail in the last few years. It is a safe bet to predict that this powerful analytical tool will find many natural application in economics and social sciences in the years to come.

As models become more realistic and hone in on details, analytics often has to give way to numerical simulations. The situation is now well accepted in physics, where numerical experimentation has gained a respectable status, which, in the words of science writer Mark Buchanan, bestows us with a "telescope of the mind, multiplying human powers of analysis and insight just as a telescope does our powers of vision".

Sadly, many economists are still reluctant to recognize that, although very far from theorem proving, numerical investigation of a model is a valid way to do science. Yet, it is a useful compass with which to venture into the wilderness of irrational-agent models: try this behavioural rule and see what comes out, explore another assumption, iterate, explore. It is actually surprising how easily these numerical experiments allow one to qualify an agent-based model as potentially realistic (on which one should dwell further) or completely off the mark.

What makes this expeditious diagnosis possible is the fact that for large systems, details do not matter much – only a few microscopic features end up surviving at the macro scale. This is a well-known story in physics: the structure of the Navier–Stokes equation for macroscopic fluid flow, for example, is independent of all molecular details. What researchers should focus on now is therefore identifying the features that explain financial markets and economic systems as we know them. This is of course still very much an open problem, and simulations will play a central role. The main drive of econophysics is that competition and heterogeneity, as described above, should be the key ingredients of a better new theory of economics.

A slew of other empirical results, useful analytical methods and numerical tricks have been established in econophysics, which I have no space to review here. But in my opinion, the most valuable contribution of physics to economics will end up being of a methodological nature. Physics has its own way of constructing models of reality based on a subtle mixture of intuition, physical analogies and mathematical treatment, where the ill-defined concept of *plausibility* can be more relevant than the accuracy of the prediction. Kepler's ellipses and Newton's gravitation were more plausible than Ptolemy's epicycles, even when the latter theory, after centuries of fixes and stitches, was initially a more accurate way to describe observations.

When Anderson first heard about the theory of rational expectations in the 1987 Santa Fe meeting, his befuddled reaction was "You guys *really* believe that?". He would probably have fallen off his chair had he heard US economist and Nobel laureate Milton Friedman's complacent viewpoint on theoretical economics: "In general, the more significant the theory, the more unrealistic the assumptions." Physicists definitely want to know what an equation means in intuitive terms, and believe that assumptions ought to be both plausible and compatible with observations. This is probably the most urgently needed paradigm shift in economics. ■

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More about: Econophysics

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